

# Walker-like Domain Wall breakdown in layered Antiferromagnets driven by staggered spin-orbit fields

## Supplementary Note 1

We revisit here the concept of the so-called gyrofield to explain the physical origin of the vortex core reversal in submicron-sized dots [1]. We extend its treatment to 1D systems and to purely translational motion. Hence, associated to  $DW_1$ , driven by SO-field, emerges a magnetic field whose origin is due the translational dynamics of the DW itself. The kinetic part of the Lagrangian density can be written in the invariant form as [1-3]

$$\mathcal{L}_{\text{kin}}(\mathbf{m}, \dot{\mathbf{m}}) = \frac{M_s}{\gamma} \frac{\mathbf{n} \cdot (\mathbf{m} \times \dot{\mathbf{m}})}{1 + \mathbf{m} \cdot \mathbf{n}}, \quad (1)$$

where  $\mathbf{m} = \mathbf{M}/M_s$  is the reduced magnetization, and  $\mathbf{n}$  is a unit vector along an arbitrary direction called as the Dirac's string,  $\gamma$  is the gyroscopic constant and  $\dot{\mathbf{m}} = \partial_t \mathbf{m}$ .

In the following, we calculate the kinetic field for a Néel domain wall moving along the  $\mathbf{x}$ -axis due to the action of a spin-orbit field directed along the  $\mathbf{y}$ -axis. It was suggested by Guslienko et al. [1] to re-write the kinetic Lagrangian (Eq. 1) in the form of the effective Zeeman energy

$$\mathcal{L}_{\text{kin}} = M_s \mathbf{m} \cdot \mathbf{h}, \quad (2)$$

where an effective magnetic field  $\mathbf{h}$  (gyrofield or kinetic field) proportional to the magnetisation derivative with respect to time  $\dot{\mathbf{m}}$  was introduced as

$$\mathbf{h} = \frac{1}{\gamma} \frac{1}{(1 + \mathbf{n} \cdot \mathbf{m})^2} [-\mathbf{m} \times \dot{\mathbf{m}} + (\dot{\mathbf{m}} \times \mathbf{n})(1 + \mathbf{n} \cdot \mathbf{m})]. \quad (3)$$

Note that only longitudinal component of the gyrofield along the  $\mathbf{n}$  direction (the first term in Eq. (3)) was used in [1] to describe critical velocity of the vortex core motion to reach the vortex core polarization reversal. The longitudinal component of the gyrofield satisfies the usual relation of the energy density and corresponding effective field

$$h_n = \frac{1}{M_s} \frac{\delta \mathcal{L}_{\text{kin}}}{\delta m_n}, \quad (4)$$

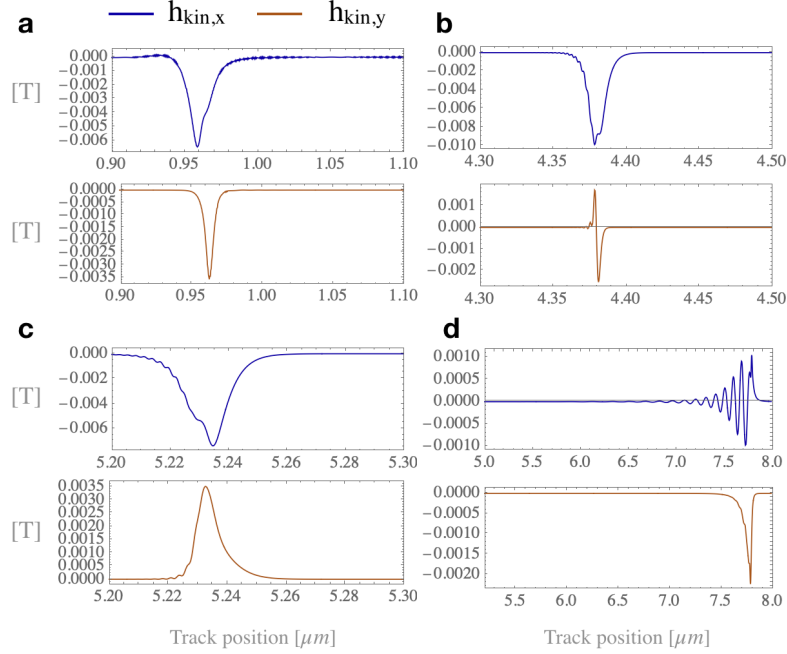
whereas for the transverse gyrofield components (perpendicular to the  $\mathbf{n}$  direction) the relation

$$\mathbf{h}_t = \frac{1}{2M_s} \frac{\delta \mathcal{L}_{\text{kin}}}{\delta \mathbf{m}_t} \quad (5)$$

is valid. The variation derivatives with respect to  $\mathbf{m}$ ,  $\dot{\mathbf{m}}$  dynamical variables are used in Eqs. (4), (5).

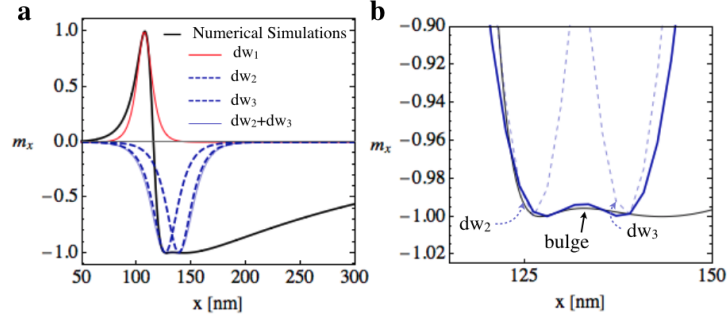
The  $\mathbf{n}$  direction was chosen along the cylindrical dot axis Oz (vortex core magnetisation direction). In the case of a domain wall dynamics and parametrisation of the magnetisation via the spherical angles  $\mathbf{m}(\theta, \phi)$  it seems that  $\mathbf{n}$  can be defined as direction of the z-axis of the spherical coordinate system. Note that the kinetic field given by Eq. 3 is related to neither the magnetic emergent field nor electric emergent field which are often introduced to describe the dynamics of magnetisation textures (vortices, skyrmions, domain walls etc.). See, for instance [4]. Nevertheless, the concept of the kinetic field is useful to describe qualitatively the dynamics of the magnetisation textures because it is proportional to the texture velocity and can be used as a fictitious driving force for the magnetisation texture dynamic deformations.

The  $\mathbf{z}$ -component of the kinetic field is found to reach values up to 35 T in absolute value, the  $\mathbf{x}$  and  $\mathbf{y}$  components reach a maximum absolute value of circa 6 mT and 3.5 mT respectively, see Supplementary Figure 1. Such big difference between the kinetic field in-plane and out-of-plane components is due to specific magnetization texture considered. The domain wall magnetisation is in the basal  $xOy$  plane and  $m_z = 0$  approximately.



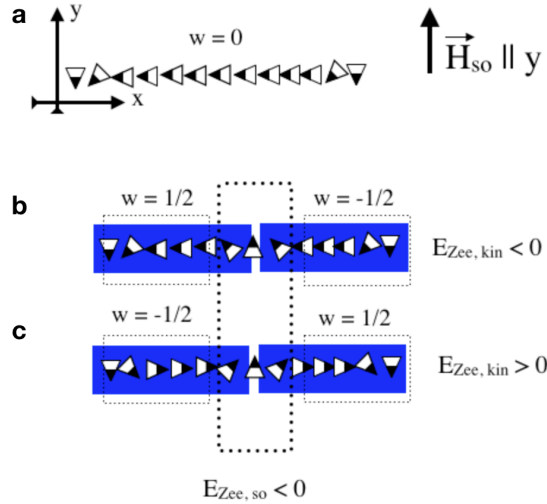
Supplementary Figure 1: Snapshots of  $x$  and  $y$ -components of the kinetic field at different instances during  $DW_1$  motion along the track. Panel **a** to **d** corresponds to physical times of 10, 90, 110 and 145 picoseconds.

The region generated in front of the  $DW_1$  that is fully reversed along the  $-m_x$  direction (see Supplementary Figure 2) is about 20 nm in the moving frame. In the rest frame this length corresponds to more than twice the exchange length,  $l_{ex}$  defined as:  $l_{ex} = (A/K_{ani})^{1/2} = 19.78$  nm, where  $A$  and  $K_{2||}$  represent the effective exchange stiffness and uniaxial anisotropy constants respectively (see Table 1 in the main text). Once this occurs, the energy competition between the anisotropy energy and the exchange energy over the region which is twice  $l_{ex}$  led to a formation of domains walls preserving the overall winding number. We note that the crucial point is not just the  $DW_1$  to reach certain speed to generate a large enough kinetic fields but also that the kinetic field extends sufficiently in space to accommodate a  $DW$ -pair.



Supplementary Figure 2: **a**  $m_x$  distribution along the track. We have over imposed a domain wall profile (black solid line) given by Eq. 5 to extract the DW<sub>1</sub> width as well as profiles for DW<sub>2</sub> and DW<sub>3</sub>. **b** Zoom-in on the region where for the first time we observe a small bulge that suggest the nucleation of DW<sub>2</sub> and DW<sub>3</sub> is about to begin. We have again super imposed a profile comprised by DW<sub>2</sub> and DW<sub>3</sub> (dashed blue lines) in order to reproduce in the best manner the small bulge. The domain wall widths extracted from the fitting for DW<sub>2</sub> and DW<sub>3</sub> are:  $\Delta_{DW_2} = \Delta_{DW_3} = 10$  nm.

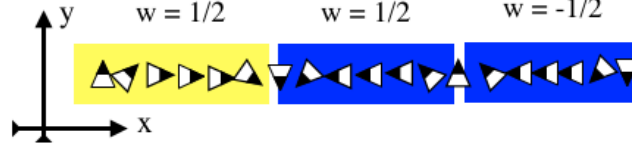
Different type of magnetic distributions could in principle appear obeying the boundary conditions imposed over the generated magnetic domain, see Supplementary Figure 3 **a**. Supplementary Figure 3 panel **b** shows an arrangement where the Zeeman energy comprised by the spin-orbit field and the kinetic field are minimised. Moreover, one can differentiate two magnetic textures whose winding number  $w$ , are opposite giving an overall winding number zero and therefore preserving the overall topological charge. Panel **c** in Supplementary Figure 3 shows that although that particular arrangement minimises the spin-orbit part of the Zeeman energy, it is not compatible with the orientation of the spins promoted by the torque exerted by the kinetic field (along negative  $\hat{x}$ .dir.) nor the Zeeman energy from the kinetic field.



Supplementary Figure 3: **a** Schematic of the coordinate system and the orientation of the spin-orbit field  $\vec{H}_{so}$ . Panels **b** - **c** corresponds to the possible arrangements compatible with the spin-orbit field but only the texture shown in **b** correspond to appropriate arrangement consistent with the torque exerted by the z-component of the kinetic field as well as the the minimisation of the Zeeman energy coming from the kinetic field and the SO-field.

## Supplementary Note 2

The topological charge distribution extracted from the numerical simulations corresponds to:  $|\text{DW}_1, \text{DW}_2, \text{DW}_3\rangle = |\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\rangle$  where the numbers within the bracket represent the winding number associated to each  $\text{DW}_i$ . As  $\text{DW}_1$  and  $\text{DW}_2$  have the same topological charge implies that the central spin of each of them are antiparallel oriented with respect to each other (as shown in Supplementary Figure 4) which implies that, as they are located in the same ferromagnetic basal plane, the exchange interaction among them will be therefore repulsive.



Supplementary Figure 4: Illustration of the magnetisation distribution after the nucleation process has been completed. This assortment as shown before is the only compatible with the conditions of the numerical simulations and will be our starting point for explaining the boost experienced by the  $\text{DW}_3$

In the following, we calculate the exchange interaction between  $\text{DW}_1$  and  $\text{DW}_2$  as a function of the distance between them. The azimuthal angle of a 1D domain wall is described by

$$\Phi(x, t) = 2 \arctan \left[ \text{Exp} \left[ q \frac{(x - X(t))}{\Delta(t)} \right] \right], \quad (6)$$

where  $X(t)$  represents the domain wall position,  $\Delta(t)$  the domain wall width and  $q$  the topological charge. Note that even though the domain wall is treated as a rigid object is subject to contraction and so  $\Delta$  has an explicit dependence on time,  $t$ . For sake simplicity, we assume two domain walls with the same width and same topological charges so that  $q_{1,2} = q = +1$ . Therefore the compound profile of the two magnetic domain wall will simply be the superposition of both profiles

$$\Phi(x) = 2 \arctan \left[ \exp \left[ q \frac{(x - X_1(t))}{\Delta(t)} \right] \right] + 2 \arctan \left[ \exp \left[ q \frac{(x - X_2(t))}{\Delta(t)} \right] \right], \quad (7)$$

The exchange interaction  $E_{\text{exc}}$ , is given by

$$E_{\text{exc}} = A \int_{-\infty}^{+\infty} \left( \sum_i \partial_x \Phi_i \right)^2 dx, \quad (8)$$

where the integral is calculated over the whole 1-dimensional chain. For a single domain wall the spatial derivative of the azimuthal angle is given by

$$\partial_x \Phi_i = \frac{q}{\Delta(t)} \text{sech} \left( q \frac{(x - X_i(t))}{\Delta(t)} \right). \quad (9)$$

Hence, the overall exchange interaction composed by the two magnetic textures is

$$\sum_i \partial_x \Phi_i = \frac{1}{\Delta(t)} \left[ \text{sech} \left( \frac{x - X_1(t)}{\Delta(t)} \right) + \text{sech} \left( \frac{x - X_2(t)}{\Delta(t)} \right) \right]. \quad (10)$$

Now just taking the square in Eq. 9 allows us to calculate the exchange interaction

$$\left(\sum_i \partial_x \Phi_i\right)^2 = \frac{1}{\Delta(t)^2} \left[ \text{sech}^2\left(\frac{x - X_1(t)}{\Delta(t)}\right) + \text{sech}^2\left(\frac{x - X_2(t)}{\Delta(t)}\right) + 2 \text{sech}\left(\frac{x - X_1(t)}{\Delta(t)}\right) \text{sech}\left(\frac{x - X_2(t)}{\Delta(t)}\right) \right], \quad (11)$$

where we can identify the first two terms as the self-exchange energy terms for each of the domain walls

$$E_i = \frac{A}{\Delta(t)^2} \int_{-\infty}^{+\infty} \text{sech}^2\left(\frac{x - X_i(t)}{\Delta(t)}\right) dx. \quad (12)$$

Whereas the third term in Eq. 10 corresponds to the interaction between the domain walls.

$$E_{12} = \frac{2A}{\Delta(t)^2} \int_{-\infty}^{+\infty} \text{sech}\left(\frac{x - X_1(t)}{\Delta(t)}\right) \text{sech}\left(\frac{x - X_2(t)}{\Delta(t)}\right) dx. \quad (13)$$

It is possible to rewrite Eq. (11) as

$$E_{\text{exc}} = E_1 + E_2 + E_{12}. \quad (14)$$

In order to calculate the interaction term,  $E_{12}$ , given by Eq. 12, we make use of hyperbolic functions and the following tabulated trigonometric relationship

$$\cosh(\xi_1) \cosh(\xi_2) = a + a \cosh(2\xi_1) + c \sinh(2\xi_1), \quad (15)$$

where  $\xi_i = (x - X_i(t)) / \Delta(t)$ , and the coefficients  $a$  and  $c$  are defined as

$$a = \frac{1}{2} \cosh\left(\frac{X_1(t) - X_2(t)}{\Delta(t)}\right), \quad (16)$$

$$c = \frac{1}{2} \sinh\left(\frac{X_1(t) - X_2(t)}{\Delta(t)}\right). \quad (17)$$

The integral in Eq. 12 can be now expressed as

$$E_{12} = \frac{2A}{\Delta^2} \int_{-\infty}^{+\infty} \frac{dx}{a + a \cosh(2\xi_1) + c \sinh(2\xi_1)} = \frac{2A}{\Delta^2} I, \quad (18)$$

where  $I$  is the sum of the two following integrals

$$I = \frac{\Delta(t)}{2} \left[ \int_0^{+\infty} \frac{dy}{a + a \cosh(y) + c \sinh(y)} + \int_0^{+\infty} \frac{dy}{a + a \cosh(y) - c \sinh(y)} \right] = \frac{\Delta}{2} [I(a, a, c) + I(a, a, -c)]. \quad (19)$$

The solution of this integral can be found in [5] and is given by

$$I(a, a, c) = \frac{1}{c} \ln\left(\frac{a+c}{a}\right). \quad (20)$$

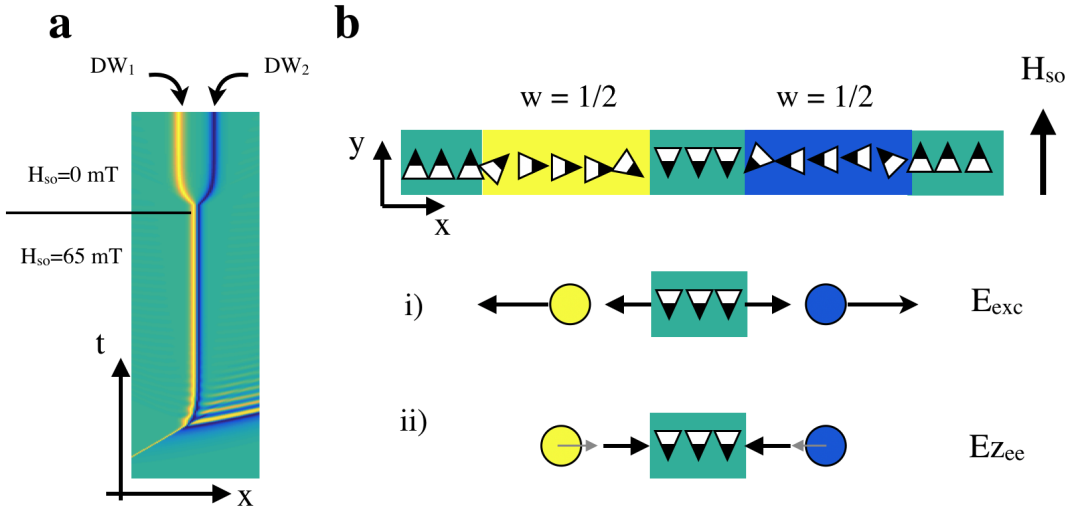
Now,  $2 \operatorname{atanh}(x) = \ln[(1+x)/(1-x)]$  and the integral for  $I$  can be rewritten as

$$I = 2(X_1(t) - X_2(t)) \operatorname{csch}\left(\frac{X_1(t) - X_2(t)}{\Delta(t)}\right), \quad (21)$$

and, subsequently, Eq. 12 can be expressed as

$$E_{12} = q_1 \cdot q_2 \cdot \frac{4A(X_1(t) - X_2(t))}{\Delta(t)^2} \operatorname{csch}\left(\frac{X_1(t) - X_2(t)}{\Delta(t)}\right), \quad (22)$$

where the product of the DW topological charges  $q_1 \cdot q_2 = +1$  corresponds to repulsion, and  $q_1 \cdot q_2 = -1$  to attraction of DWs.



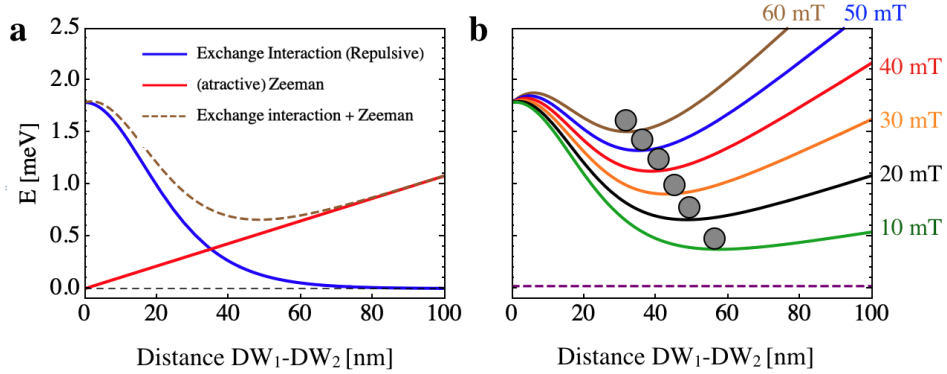
Supplementary Figure 5: **a** Temporal evolution of  $DW_1$  and  $DW_2$  after the nucleation. The two domain walls are separated by a distance that will be kept constant. When the spin-orbit field is set to zero the two domain walls separate even further. **b** Schematic illustration of the role played by the exchange energy and the Zeeman energy. While the exchange energy tries to separate  $DW_1$  and  $DW_2$ , the Zeeman energy forces them to stay as close as possible.

Once the nucleation process has terminated there is a stable distance between  $DW_1$  and  $DW_2$  as shown in Supplementary Figure 5 **a**. Moreover, we can observe that this distance increases when the spin-orbit field is set to zero. Supplementary Figure 5 **b** shows an illustration of the magnetisation distribution. We can see that the two domain walls ( $DW_1$  and  $DW_2$ ) have the same winding number and as a consequence, it appears a region in between them whose polarisation is along the  $-\hat{y}$ -direction. Sub-panel i) in Supplementary Figure 5 **b** shows that the repulsive interaction between  $DW_1$  and  $DW_2$  due to the exchange interaction,  $E_{\text{exc}}$ , results in an expansion of the magnetic domain separating both domain walls. On the contrary, ii) sub-panel shows that as the magnetic domain between the domain walls has opposite orientation with respect to the spin-orbit field,  $H_{\text{so}}$ , the Zeeman energy,  $E_{\text{zee}}$ , tries to shrink the magnetic domain in order to minimise the Zeeman energy and as a result the two domain walls are pulled together. Overall, we can see that the stable distance among the two domain walls is due to the competition between the exchange and Zeeman energies. This competition can be expressed as

$$\Delta E = (X_1 - X_2)M_s H_{\text{so}} + \frac{4A(X_1 - X_2)}{\Delta^2} \operatorname{csch}\left(\frac{X_1 - X_2}{\Delta}\right), \quad (23)$$

where as before  $X_1$  and  $X_2$  represent the location of the domain walls' center,  $\Delta$  is the domain wall width (see Supplementary Figure 2) for each domain walls extracted from the numerical simulations.

Supplementary Figure 6 **a** shows the different energies in action for a spin-orbit field of 20 mT. As the the spin-orbit field and magnetic domain between the two domain walls is antiparallel, the minimum energy corresponds to a zero distance between the walls. The opposite happens with the exchange energy, the smaller is the distance between the two domain walls the larger is the exchange energy ( $A > 0$ ). One can see that (brown dashed line) that when combining the exchange and the Zeeman energy appears a global minimum which corresponds to a stable configuration distance between  $DW_1$  and  $DW_2$ . The stable distance between the two domain walls as a function of the applied spin-orbit field is represented in Supplementary Figure 6 **b**. It can be observed that starting for the larger spin-orbit field (60 mT), the stable distance increases as the spin-orbit field (SO-field) decrease meaning that the Zeeman force needs more extension of the magnetic domain to compensate the repulsion between the domain walls.



Supplementary Figure 6: **a** Exchange and Zeeman energy as a function of the  $DW_1$  to  $DW_2$  distance for an applied spin-orbit field of 20 mT. **b** Total energy comprised by the exchange energy and Zeeman energy for different applied spin-orbit fields. The small grey dots represent the global minimum for the different spin-orbit fields and the dashed purple line represents the thermal energy ( $k_B T$ ) at room temperature.

### Supplementary Note 3

As the nucleation occurs while  $DW_1$  translates, we assume that the dip which will eventually led to the DW-pair,  $DW_2$  and  $DW_3$  is also propagating at the same velocity as  $DW_1$ . The  $DW_1$  mobility depends linearly on the SO-field as follows

$$v_{DW_1} = \frac{\gamma}{\alpha} H_{so} \Delta_0 \left[ 1 - \left( \frac{v_{DW_1}}{v_g} \right)^2 \right]^{1/2}, \quad (24)$$

where  $\alpha$  is the damping constant,  $\gamma$  is the gyromagnetic ratio,  $H_{so}$  is the spin-orbit field,  $\Delta_0$  is the DW-width at rest and  $v_g$  is the maximum group velocity of the magnons. The sole energies involved in the dynamics of  $DW_3$  are: the Zeeman energy (coming from the spin-orbit field) and the exchange interaction between  $DW_1$  and  $DW_2$  which we assume that is transferred to  $DW_3$ . The repulsive exchange energy between  $DW_1$  and  $DW_2$  is given by

$$\mathcal{E}_{exc} = \frac{4A(X_p(t_0) - X_1(t_0))}{\Delta_{DW}^2(t_0)} \operatorname{csch} \left( \frac{X_p(t_0) - X_1(t_0)}{\Delta_{DW}(t_0)} \right), \quad (25)$$

where  $X_p(t_0) - X_1(t_0)$  is the distance between  $DW_1$  and DW-pair,  $\Delta_{DW}$  is the domain wall width extracted from the simulations.  $A$  is the effective ferromagnetic exchange stiffness. For sake of

simplicity, we translate our system of reference to DW<sub>1</sub> which implies  $X_1(t_0) = 0$  and therefore  $X_p(t_0) - X_1(t_0) = x(t_0)$ . We obtain the DW<sub>3</sub> width by fitting two domain walls to the dip in front of the moving DW<sub>1</sub> as shown in Supplementary Figure 2 b.

The Lagrangian formalism allows us to characterise the DW<sub>3</sub> boost as a function of its distance to the DW<sub>2</sub>. By integrating over the Lagrangian density,  $\mathcal{L}$ , for the magnetisation profile of the Néel wall in an embedded ferromagnetic plane, we obtain the temporal evolution of each canonical coordinate,

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\xi}} - \frac{\partial \mathcal{L}}{\partial \xi} = F_\xi, \quad (26)$$

where  $F_\xi$  makes reference to non-conservative forces and can be calculate by the so-called Rayleigh dissipation function,  $W$

$$F_\xi = -\frac{\partial W}{\partial \dot{\xi}} = -\frac{\alpha \mu_0 M_s}{2\gamma} \frac{\partial}{\partial \dot{\xi}} \int d^3r \left( \dot{\Phi}^2 + \dot{\Theta}^2 \sin^2 \Phi \right), \quad (27)$$

where  $\mu_0$  is the magnetic permeability in vacuum and  $\Phi$  and  $\Theta$  represents the in-plane and out-of-plane component of the magnetisation. Here after, we assume that the values for the damping, saturation magnetisation, anisotropy constants and ferromagnetic exchange stiffness are the same for all the ferromagnetic layers that conform the Mn<sub>2</sub>Au unit crystal.

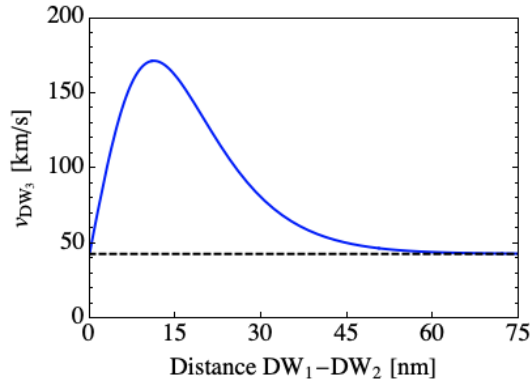
By combining again the Rayleigh dissipation function and the Euler-Lagrange equation of motion allows us to get the boost over  $v_{\text{DW}_3}$  as a function of its distance to DW<sub>1</sub>

$$v_{\text{DW}_3} = \frac{2\gamma A}{\alpha M_s \Delta_{\text{DW}_3}^2} \left[ x(t_0) \coth \left( \frac{x(t_0)}{\Delta_{\text{DW}_3}(t_0)} \right) - \Delta_{\text{DW}_3}(t_0) \right] \text{csch} \left( \frac{x(t_0)}{\Delta_{\text{DW}_3}(t_0)} \right). \quad (28)$$

However, the overall DW<sub>3</sub> speed will be comprised by its initial velocity provided by the DW<sub>1</sub> and the boost coming from the repulsion between DW<sub>1</sub> and DW<sub>2</sub> which can be expressed as

$$v_{\text{DW}_3} = \frac{\gamma}{\alpha} H_{\text{so}} \Delta_0 \left[ 1 - \left( \frac{v_{\text{DW}_1}}{v_g} \right)^2 \right]^{1/2} + \frac{2\gamma A}{\alpha M_s \Delta_{\text{DW}_3}^2} \left[ x(t_0) \coth \left( \frac{x(t_0)}{\Delta_{\text{DW}_3}(t_0)} \right) - \Delta_{\text{DW}_3}(t_0) \right] \text{csch} \left( \frac{x(t_0)}{\Delta_{\text{DW}_3}(t_0)} \right). \quad (29)$$

Supplementary Figure 7 shows the dependence of the maximum speed experienced by DW<sub>3</sub> as a function of the distance between DW<sub>3</sub> and DW<sub>1</sub> according to Eq. 28. Note, the maximum boost does not correspond to a zero distance due to finite size of the DWs. Instead, the maximum boost in speed which corresponds to 133 km/s appears at a distance of 16.1 nm between the DW<sub>1</sub> and DW<sub>3</sub>. From numerical simulations, the distance between the DW<sub>1</sub> and DW<sub>2</sub> is 17 nm and the extracted value for the speed about 177 km/s.



Supplementary Figure 7: Blue color line represents the DW<sub>3</sub> velocity as a function of the distance between DW<sub>1</sub> and DW<sub>3</sub>. Dashed black line represents the maximum group velocity of the magnons,  $v_g$ .

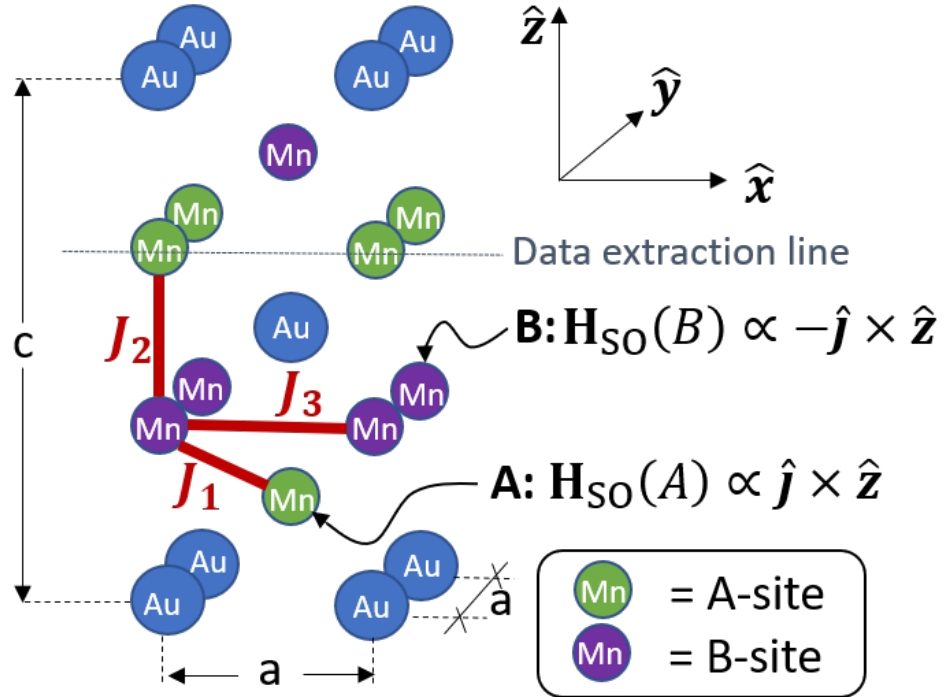


## Supplementary Note 4

A  $\text{Mn}_2\text{Au}$  spin lattice is used for the atomistic spin dynamics simulations. The structure is shown in Supplementary Figure 8. There are two types of sites occupied by Mn, A and B as indicated in the figure. A current density along unit-direction  $\hat{\mathbf{j}}$  coinciding with the  $x$ -direction, results in a staggered magnetic field,  $\mathbf{H}_{\text{SO}}$  at each site (see Supplementary Figure 8).

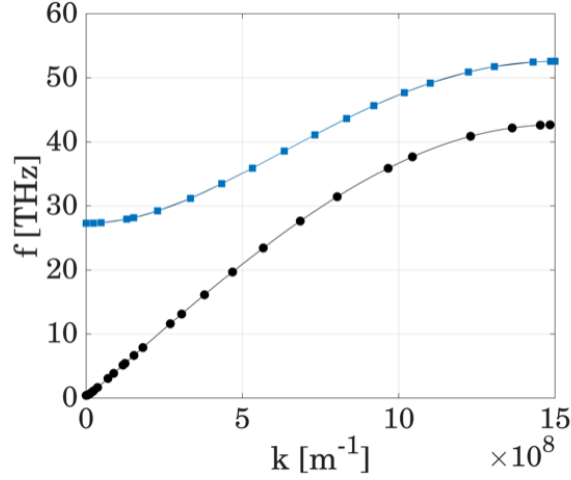
We use 60000 unit  $\text{Mn}_2\text{Au}$  lattice cells in the simulations along  $x$ . Periodic boundary conditions (wrap-around) are applied along the  $y$ -direction. As initial condition for the dynamical simulations we use precomputed and stabilized  $180^\circ$  domain walls. Spins in the  $xy$ -plane are ferromagnetic ally ordered (the  $J_3$ -bonds) and antiferromagnetically coupled along  $z$  (the  $J_1$  and  $J_2$  -bonds) as also indicated in Supplementary Figure 8). Data is extracted along a 1D line shown by a dotted line in Supplementary Figure 8 denoted Data extraction line.

The LLG-equation is integrated using a 5th order Runge-Kutta scheme with adaptive time-stepping. The domain wall position is tracked from moments of the winding number density  $w$  along the data extraction line at each time  $t$  via  $x_{\text{DW}}(t) = \frac{\int x w(x,t) dx}{\int w(x,t) dx}$  from which the speed is computed. The time dependence of the DW width is computed by fitting the  $S_y$ -component along the data extraction line to  $\phi(x,t) = 2 \arctan \left[ \exp \left( \frac{x-x_{C,t}}{\Delta_{\text{DW}}(t)} \right) \right]$ , in which  $x_{C,t}$  is the DW centre and  $\Delta_{\text{DW}}(t)$  is the DW width. The fitted values will yield a slight overestimation of the actual width during acceleration and deceleration stages of the DW motion as the fitting function assumes a symmetric profile.



Supplementary Figure 8: Single  $\text{Mn}_2\text{Au}$  cell used in the simulations. Mn-spins occupy A and B-type sites resulting in a staggered distribution of  $\mathbf{H}_{\text{SO}}$  as indicated in this figure. The exchange bonds taken into consideration,  $J_1$ ,  $J_2$  and  $J_3$  are shown by thick red lines. Note that the ferromagnetic  $J_3$  bond acts between all nearest neighbour's in the  $xy$ -plane. Crystal dimensions are  $a=0.3328$  nm and  $c = 0.8539$  nm. The 1D line whereby spatio-temporal data is extracted from simulations is shown as a finely dotted line and denoted " Data extraction line ".

The maximum magnon group velocity  $v_g$  is estimated in a separate simulation according to the following steps: A homogeneous Néel state is used as starting state. Then spins within a local region in the centre are slightly perturbed by imposing small  $S_x$ -component. The symmetry of this perturbation is such that the induced  $S_x$ -components alternate in sign along the  $z$ -direction. Using the perturbed state as starting configuration, the system is let to ring down and the time dependence of  $S_x$  is recorded at each site on the data extraction line. In a last step, a 2D FFT is performed, giving frequency ( $f$ ) versus k-vector along  $x$  ( $k_x$ ). The  $(f, k_x)$  points with maximum oscillation-amplitudes is then extracted to form a line plot of the dispersion relation shown in Supplementary Figure 9.



Supplementary Figure 9: Computed dispersion relation. The lower branch is used in the velocity vs. k-vector plot in the main text as it contains the highest slope and thus yields the maximum group velocity.

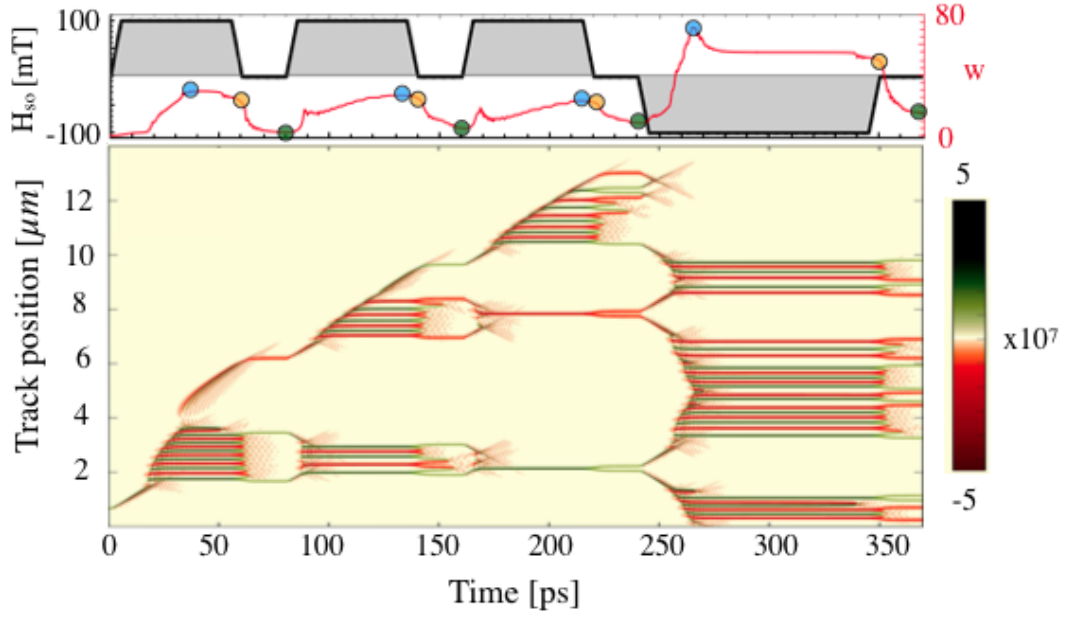
The maximum magnon group velocity  $v_g$  is extracted from the lower branch (black filled circles and black line in Supplementary Figure 9) by  $v_g = \left( \frac{d\omega(k)}{dk} \right)_{\max}$ , where  $\omega = 2\pi f$ . The dispersion-relation will be affected by the presence of a background SO-field present when the DW is being driven. However, the effect is only to shift the dispersion relation along the frequency-axis does not alter  $\left( \frac{d\omega(k)}{dk} \right)$ . The relation in Supplementary Figure 9 is obtained under a constant  $\mathbf{H}_{\text{so}} = 100$  mT. For the calculation of the dispersion relation, we used a shorter computational domain (40000 cells long) due to resulting large file sizes due to fine time sampling. The parameters were: time step = 1 fs and data was recorded every 3 fs during a total time of 50 ps.

## Supplementary Note 5

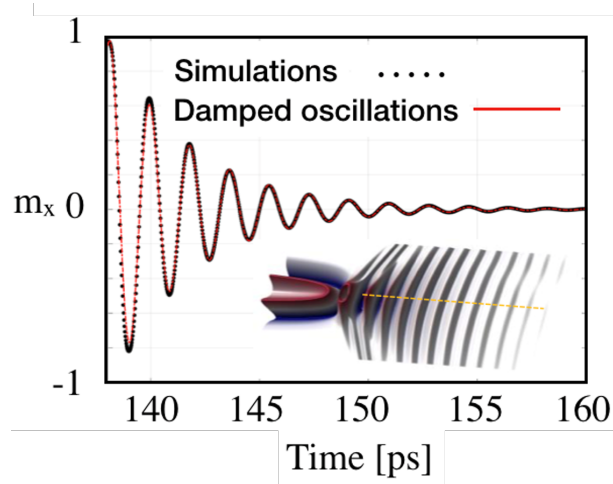
As mentioned in the main text, a different excitation protocol can result in a completely different DW distribution. Supplementary Figure 10 shows another such protocol.

## Supplementary Note 6

The dampening process of the oscillations (see Supplementary Figure 11 and its caption) can be mapped into damped harmonic oscillator reproducing nicely the convoluted oscillations obtained from numerical simulations.



Supplementary Figure 10: Top: an excitation protocol consisting of three positive pulses and one long negative pulse. Bottom, the winding number density versus space and time



Supplementary Figure 11: Time evolution of the  $m_x$ -component at a fixed point  $X_0$  of annihilation, extracted along the  $(X_0, t)$  line shown by the dotted yellow line in the inset. The oscillations are well fitted by a function  $y = \text{Exp}(-bt)[A \cdot \cos(\omega t - \phi)]$ , thus verifying the exponential decay in Eq. 30

## Supplementary References

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